Radical Equations

In solving equations with radicals, there are two things to keep in mind.

1) You will need to remove the radical. This can usually be done by isolating the radical and squaring or cubing both sides of the equation.

2) Be careful that solutions are real

Examples:

$$\sqrt{x} - 8 = 0$$
$$\sqrt{x} = 8$$
$$\left(\sqrt{x}\right)^2 = 8^2$$
$$x = 64$$

Check

$$\sqrt{64} - 8 = 8 - 8 = 0$$

$$\sqrt{3x} + 6 = 0$$
$$\sqrt{3x} = -6$$
$$\left(\sqrt{3x}\right)^2 = \left(-6\right)^2$$
$$3x = 36$$
$$x = 12$$

Check

$$\sqrt{3 \cdot 12} + 6 = \sqrt{36} + 6 = 6 + 6 = 12 \neq 0$$

So there is no solution

Example:

$\sqrt[3]{2x+1} - 2 = 3$

$\sqrt[3]{2x+1} = 5$

$\left(\sqrt[3]{2x+1}\right)^3 = 5^3$

$2x + 1 = 125$

$2x = 124$

$x = 62$

Check

$\sqrt[3]{2 \cdot 62 + 1} - 2 = \sqrt[3]{125} - 2 = 5 - 2 = 3$

Two Radicals

$$\sqrt{5x+1} = \sqrt{x+11}$$
$$\left(\sqrt{5x+1}\right)^2 = \left(\sqrt{x+11}\right)^2$$
$$5x+1 = x+11$$
$$4x = 10$$
$$x = 5/2$$

Check

$$\sqrt{5\cdot\frac{5}{2}+1} = \sqrt{\frac{5}{2}+11}$$
$$\sqrt{\frac{25}{2}+\frac{2}{2}} = \sqrt{\frac{5}{2}+\frac{22}{2}}$$
$$\sqrt{\frac{27}{2}} = \sqrt{\frac{27}{2}}$$

$$\sqrt[4]{3x} + \sqrt[4]{2x-5} = 0$$
$$\sqrt[4]{3x} = -\sqrt[4]{2x-5}$$
$$\left(\sqrt[4]{3x}\right)^4 = \left(-\sqrt[4]{2x-5}\right)^4$$
$$3x = 2x-5$$
$$x = -5$$

Check

$$\sqrt[4]{3(-5)} + \sqrt[4]{2(-5)-5} = \sqrt[4]{-15} + \sqrt[4]{-15} \neq 0$$

so no solution

Substituting to create a quadratic equation

$$\sqrt{x} + 2 = x$$

substitute $y = \sqrt{x} \rightarrow y^2 = x$

$$\sqrt{x} + 2 = x$$
$$y + 2 = y^2$$
$$y^2 - y - 2 = 0$$
$$(y-2)(y+1) = 0$$

So $y=2$ and $y=-1$ or $x=4$ and $x=1$ are possible solutions.

Check

$$\sqrt{4} + 2 = 2 + 2 = 4$$

$$\sqrt{1} + 2 = 1 + 2 = 3 \neq 1$$

So only 4 is a solution

Repeated squaring

$$\sqrt{3x+1} = 2 - \sqrt{3x}$$

$$\left(\sqrt{3x+1}\right)^2 = \left(2 - \sqrt{3x}\right)^2$$

$$3x + 1 = 4 - 4\sqrt{3x} + 3x$$

$$4\sqrt{3x} = 3$$

$$\sqrt{3x} = \frac{3}{4}$$

$$\left(\sqrt{3x}\right)^2 = \left(\frac{3}{4}\right)^2$$

$$3x = \frac{9}{16}$$

$$x = \frac{3}{16}$$

Check

$$\sqrt{3\left(\frac{3}{16}\right)+1} = 2 - \sqrt{3\left(\frac{3}{16}\right)}$$

$$\sqrt{\frac{9}{16}+1} = 2 - \sqrt{\frac{9}{16}}$$

$$\sqrt{\frac{25}{16}} = 2 - \sqrt{\frac{9}{16}}$$

$$\frac{5}{4} = 2 - \frac{3}{4}$$

$$\frac{5}{4} = \frac{5}{4}$$